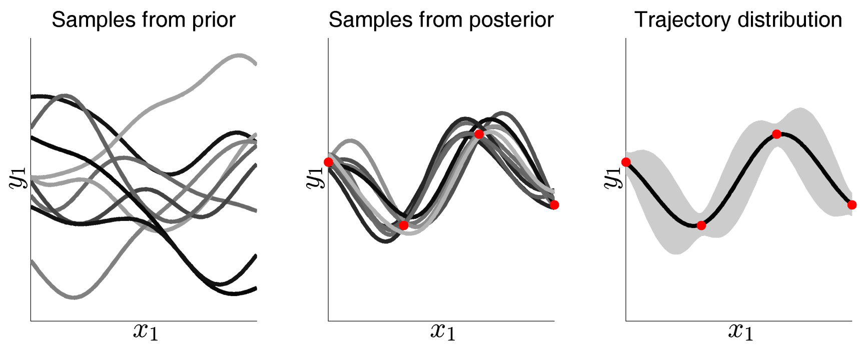
EE613 - Nonlinear regression - Exercises - Nov. 11, 2015

The main folder contains four examples `demo_LWR01.m`, `demo_GMR01.m`, `demo_GMR_polyFit01.m` and `demo_GPR01.m`. These codes can be run either from Matlab or from GNU Octave. First run the examples, visualize the results and try to change the parameters.

Exercise 1: Locally weighted regression *Vs* Gaussian mixture regression

- Modify `demo_LWR01.m` and `demo_GMR_polyFit01.m` so that it loads the dataset contained in `data/1.mat`, corresponding to 2D movement recordings to draw the digit "1".
- Set the parameters `nbStates=2` and `nbVarIn=1` in `demo_LWR01.m` and `demo_GMR_polyFit01.m`.
- Run the two examples and observe the results. Can you explain the difference?

Exercise 2: Sampling of prior and posterior distributions in a Gaussian process



The source code `demo_GPR01.m` provides an example of Gaussian process regression with a radial basis function kernel. The hyperparameters Θ_1^{GP}, Θ_2^{GP} and Θ_3^{GP} are respectively related to the scale of the output, scale of the input, and expected noise on the observed outputs.

- After making a copy of `demo_GPR01.m`, replace the dataset $\{\boldsymbol{x}, \boldsymbol{y}\}$ with four datapoints of 2 dimensions (1D input and 1D output), uniformly spread in the input dimension and random in the output dimension, as the red datapoints in the figure above. Define a list of new inputs $\boldsymbol{x}^*$ as 100 datapoints uniformly spread in the input dimension within the same range as the red points.
- By using the covariance matrix $\boldsymbol{K}(\boldsymbol{x}^*, \boldsymbol{x}^*)$ of the Gaussian process, generate stochastic samples from the prior distribution $\boldsymbol{y}^* \sim \mathcal{N}\big(\boldsymbol{\mu}(\boldsymbol{x}^*), \boldsymbol{K}(\boldsymbol{x}^*, \boldsymbol{x}^*)\big)$, by considering $\boldsymbol{\mu}(\boldsymbol{x}^*) = 0$.
- Generate stochastic samples from the posterior distribution $\boldsymbol{y}^*|\boldsymbol{y} \sim \mathcal{N}(\boldsymbol{\mu}^*, \boldsymbol{\Sigma}^*)$ of the Gaussian process, by considering $\boldsymbol{\mu}(\boldsymbol{x}) = \boldsymbol{\mu}(\boldsymbol{x}^*) = 0$.
- Test this generation procedure with different hyperparameters. An example with $\Theta_1^{\text{GP}} = 1$, $\Theta_2^{\text{GP}} = 0.1$ and $\Theta_3^{\text{GP}} = 0.01$ is given in the figure above.

Exercise 3: Gaussian process regression with periodic kernels

Based on the code of *Exercise 2*, replace the radial basis function

$$k(\boldsymbol{x}_i, \boldsymbol{x}_j) = \Theta_1^{\text{GP}} \exp\left(-\frac{1}{\Theta_2^{\text{GP}}} \, |\boldsymbol{x}_i - \boldsymbol{x}_j|^2\right) + \Theta_3^{\text{GP}} \delta_{i,j}$$

with the periodic kernel function

$$k(\boldsymbol{x}_i, \boldsymbol{x}_j) = \Theta_1^{\text{GP}} \exp\left(-\frac{1}{\Theta_2^{\text{GP}}} \, \sin^2\big(\Theta_4^{\text{GP}} |\boldsymbol{x}_i - \boldsymbol{x}_j|\big)\right) + \Theta_3^{\text{GP}} \delta_{i,j}.$$

- What do you observe when you retrieve new datapoints outside the range of the input data?